CERN
CH-1211 Geneva 23
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the **Large Hadron Collider** project

LHC Project Document No.
LHC-LQX-ES-0007 rev 1.1

CERN Div./Group or Supplier/Contractor Document No.
AT/MTM & EST/SU

EDMS Document No.
367802

Date: 2004-02-25

Engineering Specification

DEFINITIONS OF SURVEY AND MAGNETIC DATA FOR THE INNER TRIPLET SYSTEMS AT IR1, 2, 5 AND 8

Abstract

The acceptance of the inner triplet systems at interaction regions 1, 2, 5 and 8 provided within the scope of the US-LHC collaboration (the Q1-Q3 optical elements) foresees that survey and magnetic data is provided to CERN. This specification establishes the conventions and references to be used to represent the data in order to make them uniquely defined for acceptance, installation and LHC operation purposes.

Prepared by :	*Checked by :*	*To be approved by:*
Luca BOTTURA Luca.Bottura@cern.ch **Dominique MISSIAEN** Dominique.Missiaen@cern.ch	**J. Di Marco** **S. Fartoukh** **M. Lamm** **R. Ostojic** **J.P. Quesnel,** **W. Scandale** **P. Schlabach** **G. Velev**	**O. Bruning** **C. Hauviller** **S. Myers** **N. Siegel** **J. Strait** **T. Taylor**

History of Changes

Rev. No.	*Date*	*Pages*	*Description of Changes*
0.1	2003-01-09	All	Submission for check
0.2	2003-02-21	All	Submission for approval
1.0	2003-03-27	All	Released version
1.1	2004-02-12	All	Changed the position of the BPMS (warm BPM installed at CERN) in schematic drawings Changed the topology of the corrector packages MQSX and MCBXA in LQXC, adding MCSOX Removed BPMS on Q1 (mounted at CERN external to the LQXA assembly)
	2004-02-25	All	Released version

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1. INTRODUCTION

This note provides definitions for the magnetic and survey data measured on the Q1, Q2 and Q3 inner triplet systems for the LHC interaction regions 1, 2, 5 and 8. The aim of the definitions reported here is to avoid any ambiguity in the magnetic and survey data, and to make them directly usable for the qualification process, the installation and the operation of the LHC.

Table 1.1 Naming of components for the superconducting inner triplet systems

Optics Name	Main Quadrupole Code	Corrector Packages Codes	Magnet Code	Cryo-Assembly Code
Q1	MQXA	MCBX	LMQXA	LQXA
Q2	MQXB	MCBX	LMQXB	LQXB
Q3	MQXA	MCBXA MQSX MCSOX	LMQXC	LQXC

Reference naming of the magnetic elements assembled in the cold masses of the Q1, Q2 and Q3, and the Cryo-assembly code are reported in Tab. 1.1, reported from [2], [4]. The schematic layout of the inner triplet system is shown in Fig. 1.1.

All definitions in this note apply to single cryomagnets, so that the reference frames for the magnetic and survey data is independent of the location in the LHC ring. The orientation of the magnet is defined through reference to the lead end and the non-lead end of the assembled cold mass, i.e. at the level of the LMQXA, LMQXB and LMQXC components. This is the most convenient reference frame for magnet measurement. All coordinate frame transformations from the reference frames defined in this note to the reference frames used for installation and operation will be performed by CERN.

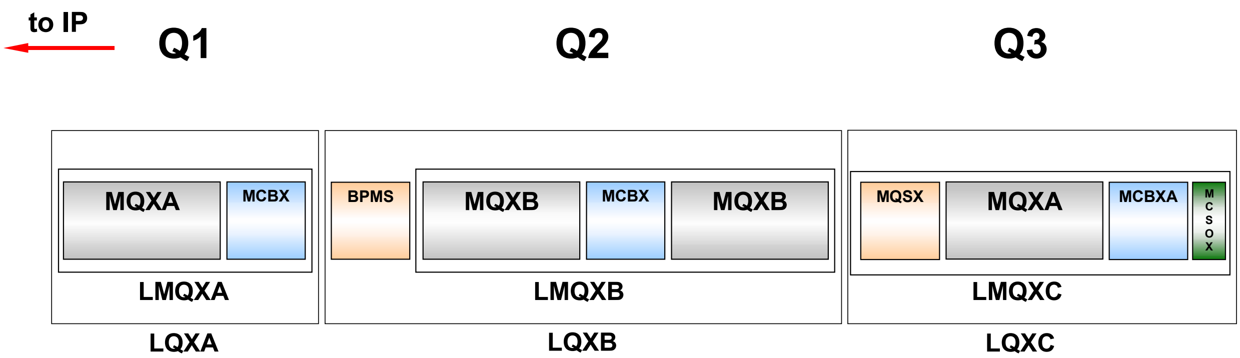

Figure 1.1. Inner triplet system schematic, from [2], [4].

2. Q1

The Q1 (LQXA) optical element contains a quadrupole MQXA whose cross section is shown in Fig. 2.1. The MQXA has a nominal magnetic length (in cold conditions) of 6.37 m. Q1 is equipped with a MCBX corrector package (on the lead end side). The lead end of Q1 is located away from the IP. See Fig. 2.2 for a schematic of the Q1 configuration and the location of the lead and IP ends. The cryostat of Q1 is equipped with eight fiducial nests for Taylor-Hobson fiducial spheres. Four of these fiducials are used for alignment, while the other four will house instrumentation for monitoring the position of the cryo-assembly after installation.

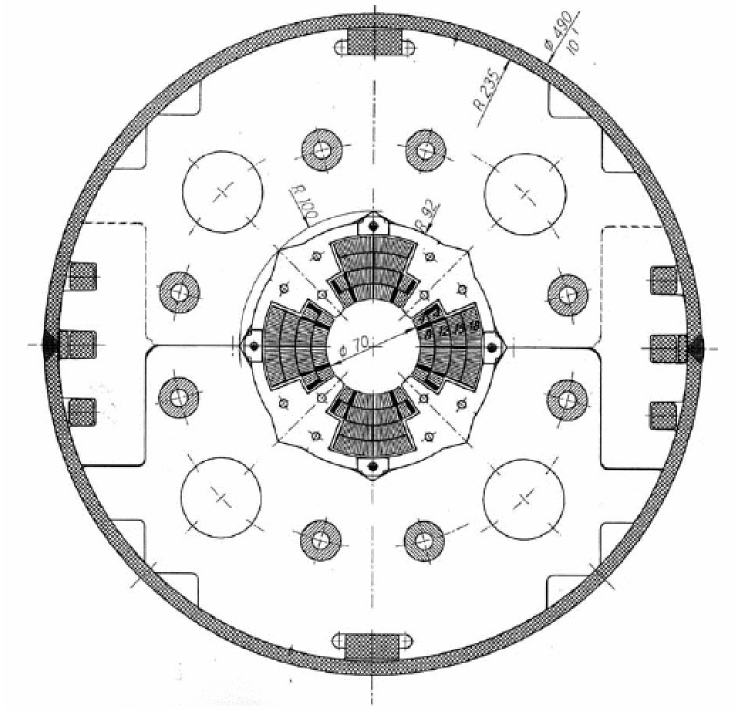

Figure 2.1. Cross section of the MQXA quadrupole, from [2].

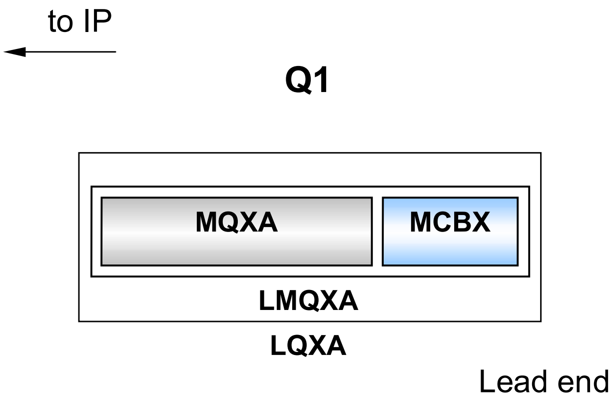

Figure 2.2. Schematic composition and location of the lead and IP ends for the LQXA cryo-assembly (Q1), from [2].

2.1 SURVEY DATA FOR Q1

Survey data for the Q1 optical element are obtained during the measurement of the magnetic axis of the MQXA magnetic element in warm conditions. The reference frame for survey data is the triad RST shown schematically in Fig. 2.3. The plane RS is the *magnetic midplane* of the magnet (defined later in this section). The S axis is the *average magnetic axis* of the magnet (defined later in this section), pointing in the direction from the lead end to the non-lead end. The R axis is oriented to the right when looking at the magnet from the lead end. This direction corresponds to the inwards direction of the LHC (pointing towards the centre of the accelerator) for the Q1 magnets installed on the *left* of the IP in the sense of Beam 1 (turning clockwise in the LHC). The T axis is normal to the RS plane, and is oriented so that the triad RST forms a right-handed reference frame.

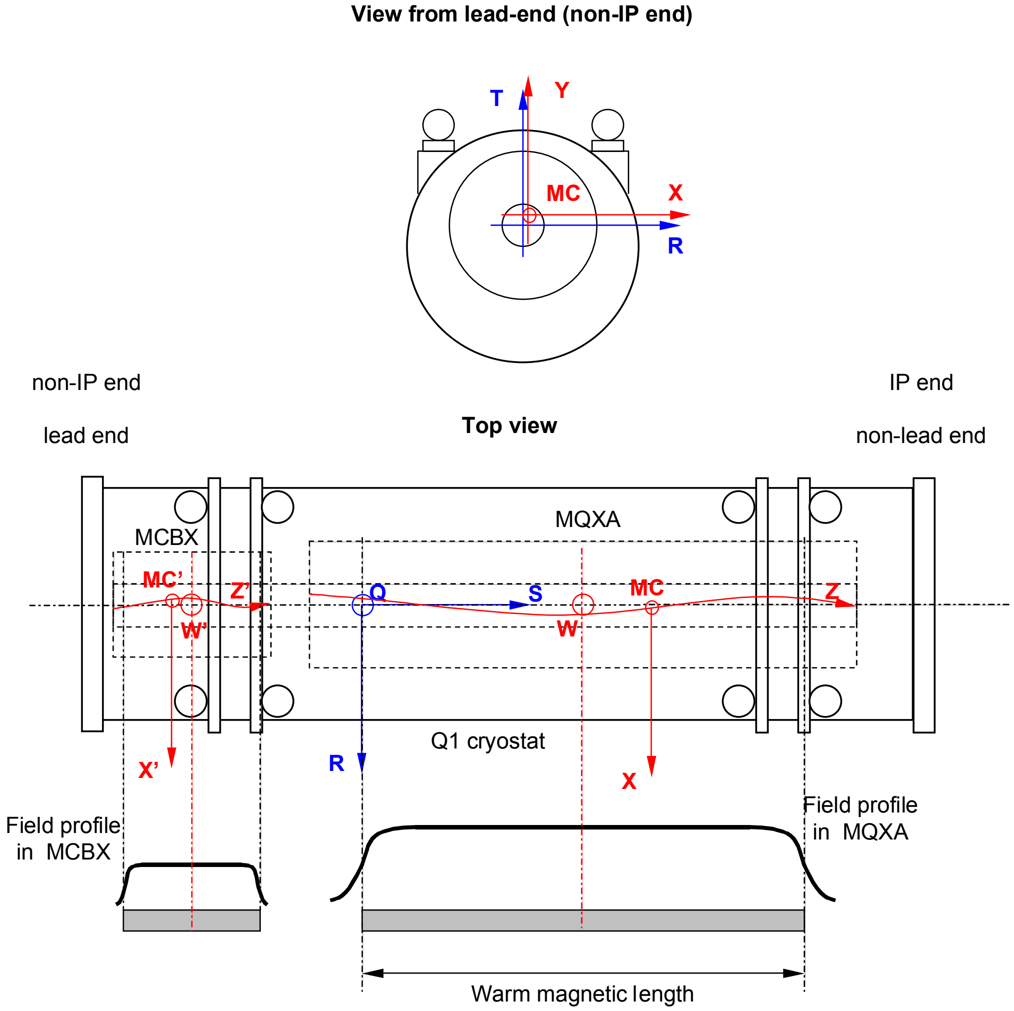

Figure 2.3. Definition of the reference frame for magnetic and alignment data of Q1. The fiducials positions on the cryostat are shown schematically. The position of points Q, the origin of the RST frame, W and W', the origins for magnetic data, is determined in warm conditions.

The *average magnetic axis* is the straight line such that the weighted average of the locus of the centers of the main quadrupole field (magnetic centers) along the MQXA quadrupole is zero. The weight for the average is the quadrupole field strength.

The *magnetic midplane* is defined as the plane normal to the average field direction measured in the MQXA quadrupole, and containing the magnetic axis S. The average field direction is defined in Appendix II

The origin of the RST frame is located at the point Q that corresponds to the beginning of the *magnetic length* of the MQXA, on the lead end, in warm conditions. The magnetic length is defined in Appendix II.

In "as built" magnets the average magnetic axis, the magnetic midplane, the RST reference frame and the position of Q are obtained directly during magnetic measurements, in warm conditions.

The survey data consists in a set of coordinates of the magnet fiducials (located on the cryostat) in the RST reference frame. The format of this data is defined in [9].

The results on magnets tested at cold (a sample of the whole production), as well as computed values, are used to establish the correlation between warm measurements and cold (operating) conditions. The warm-to-cold correlation is defined through the change in the position of the fiducials in the RST frame.

2.2 MAGNETIC DATA FOR Q1

2.2.1 MQXA QUADRUPOLE

The reference frame for magnetic data of the MQXA quadrupole in Q1 is the XYZ triad shown in Fig. 2.3. The Z axis is the *magnetic axis* of the quadrupole (see the definition given later) and the X axis points right when looking at the magnet from the lead-end. The X axis is parallel to the RS plane, as defined for the survey data. The Y axis is normal to the RS plane, and is oriented so that the diad XY forms a right-handed reference frame. The origin W of the XYZ frame is located in the longitudinal magnetic center of the cold mass, i.e. the point along the Z axis such that the field integral from the magnet connection end to W is equal to the field integral from W to the magnet non-connection end. The origin W is hence determined based solely on magnetic measurements. This definition applies both to warm and cold measurements.

The Z axis (magnetic axis) is defined as the locus of the magnetic centers MC measured along the magnet length. The magnetic center MC is defined as the point in the magnet cross section where the dipole component of the field is zero. An operative definition of the magnetic axis is given in [10]. In the ideal case of perfect dimensions and assembly, the Z axis is coincident with the average magnetic axis S. In "as built" magnets the Z axis can deviate from this theoretical position.

The magnetic data consists of local and integrated main quadrupole strength and transfer function, magnetic length, local and average field direction, local and average higher order harmonics, local and average magnetic center coordinates. All magnetic data are reported for the situation when the magnet current is "positive", i.e. entering terminal A.The definition of the local value of the field and harmonics, including the notation and conventions referred to the local XY plane, is given in [11], [10].

The local main field for a quadrupole is indicated with B_2, and corresponds to the strength (with sign) of the second complex component of the harmonic expansion, indicated with $\mathbf{C}_2$. B_2 is given in units of [T @ R_{ref}] where R_{ref} is the reference radius. The reference radius for the LHC magnets is 17 mm, but a different value can be used if convenient (e.g. in case of very low numerical values of the field harmonics) provided they it is reported in the magnetic data tables.

The local transfer function T_2 for the main quadrupole field is the ratio of the main field strength to the operating current. The transfer function T_2 is given in units of [T @ Rref/A].

The local field direction α_2 is given with respect to the Y axis, i.e. with respect to the normal T to the mechanical midplane of the magnet, in units of [radians]. The sign convention for the angle is the trigonometric positive direction (counter-clockwise). A physical rotation of the magnet by an angle $-\alpha_2$ would align the field to the Y axis. See [10] for the details on the procedure to be followed to obtain the field direction. Note that following the definitions of [10] the field direction for a quadrupole is bounded between $-\pi/4$ and $\pi/4$.

The higher order local harmonics of order n, normal b_n and skew a_n are reported in a xy reference frame with the y axis aligned with $\mathbf{C}_2$, i.e. they are obtained from the measured values after a rotation by α_2, as defined in [10]. The local harmonics b_n and skew a_n are given in relative units of 10^{-4} of the main field strength at the reference radius.

The format for the local harmonics and local field direction tables is specified in [12] and [13].

The local magnetic center coordinates are defined as the two components of the local vector between the S axis and the points on the Z axis. The magnetic center coordinates are reported in the survey frame RST.

The format for local magnetic center table is specified in [14].

The definition of the integral and average quantities based on local field measurement of the field (at several locations in the magnets) is given in Appendix II.

The integrated quadrupole strength B_2dl includes the end contributions. The integrated quadrupole strength B_2dl is given in units of [T m @ R_{ref}].

The integrated transfer function T_2dl is given in units of [T m @ R_{ref} /A].

The magnetic length L_m is given in units of [m].

The average quadrupole field direction $<\alpha_2>$ is given with respect to the normal to the magnetic midplane of the magnet, T, in units of [radians]. Note that because of the definition of the RST reference frame, the average field direction should be zero at the reference conditions used for the fiducialisation.

The average higher order harmonics of order n, normal $<b_n>$ and skew $<a_n>$ are given in relative units of 10^{-4} of the main quadrupole field strength at the reference radius.

The format for the integrated transfer function and harmonics is specified in [15] and [16].

2.2.2 MCBX CORRECTOR

The MCBX corrector contains two co-axial windings generating a normal and a skew dipole. It is assumed in this note that the two co-axial windings are aligned longitudinally and transversally in accordance with the manufacturing drawings, and the only error considered is a relative angle misalignment. The reference frame for magnetic data of the MCBX dipole corrector in Q1 is the triad X'Y'Z' shown in Fig. 2.3. The Z' axis is the *magnetic axis* of the dipole (see the definition given later). The X' and Y' axes are defined as for the MQXA quadrupole, i.e. respectively parallel and normal to the RS plane.

The Z' axis (magnetic axis of MCBX) is defined as the locus of the magnetic centers MC' measured along the magnet length. The magnetic center MC' is defined as the point in the magnet cross section where a non-allowed multipole of high order (typically order 10 or 12) is zero. The actual choice of the non-allowed multipole is inessential.

The origin of the X'Y'Z' frame is located in the longitudinal magnetic center of the dipole corrector, i.e. the point W' along the Z' axis such that the field integral from one magnet end to W' is equal to the field integral from W' to the other magnet end. Note that with the hypothesis of perfect relative longitudinal alignment of the two windings the location of the point W' can be measured indifferently on any of the two corrector windings.

The magnetic data for the MCBX corrector consists in the integrated field strength of the two windings B_1dl and A_1dl, the magnetic length of the two windings L_{mB1} and L_{mA1}, the average field direction of the two windings $<\alpha_{1B1}>$ and $<\alpha_{1A1}>$, and the longitudinal position of the magnetic element with respect to the main quadrupole MQXA (assumed to be identical for the two corrector windings).

The integrated strength B_1dl and A_1dl, including the end contributions, are given in units of [T m].

The magnetic lengths of the two windings L_{mB1} and L_{mA1} are given in units of [m].

The average field directions for the two windings $<\alpha_{1B1}>$ and $<\alpha_{1A1}>$ are reported in the X'Y' reference frame. As by definition the X' and Y' axes are parallel and normal to the RS plane, the average field directions of the MCBX dipole corrector windings are hence given with respect to the normal of the magnetic midplane of the MQXA quadrupole, T.

The longitudinal position of the dipole corrector is defined as the distance between the longitudinal magnetic centers W and W', measured along the S axis.

3. Q2

The Q2 (LQXB) optical element contains two quadrupoles MQXB whose cross section is shown in Fig. 3.1. The MQXB has a nominal magnetic length (in cold conditions) of 5.5 m. The MQXB quadrupoles are identified as Q2a, on the IP end, and Q2b, on the non-IP end of the assembly.

Q2 is equipped with a MCBX corrector package (placed between the MQXB quadrupoles) and a BPM (on the IP end side). The lead ends of the two MQXB quadrupoles are pointing in opposite directions. See Fig. 3.2 for a schematic of the Q2 configuration and the location of the lead and IP ends. The Q2 cold mass is mounted in a single cryostat.

The cryostats of Q2 is equipped with twelve fiducial nests for Taylor-Hobson fiducial spheres. Six of these fiducials are used for alignment, while the other six will house instrumentation for monitoring the position of the cryo-assembly after installation.

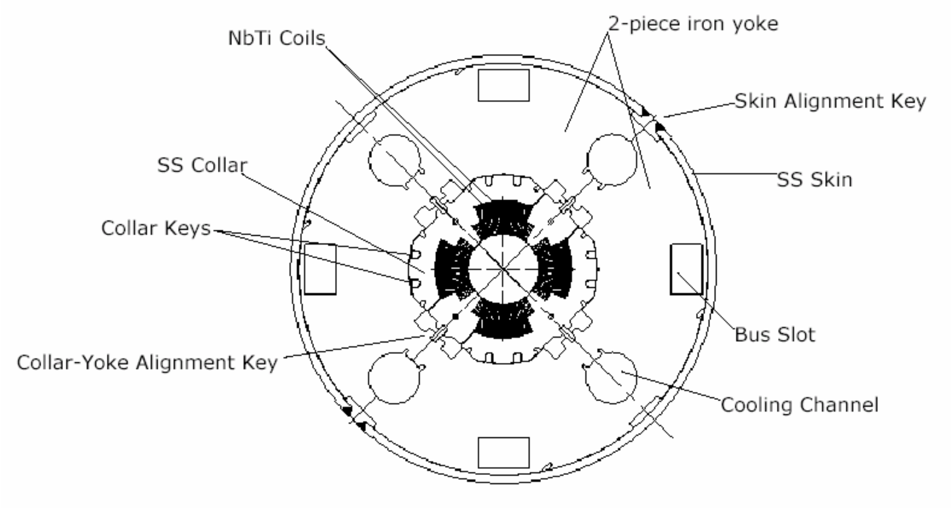

Figure 3.1. Cross section of the Q2 quadrupole, from [1], as seen facing the lead end. Note the direction pointing towards the ring center for the magnet installed on the left of the IP (pointing right in the figure).

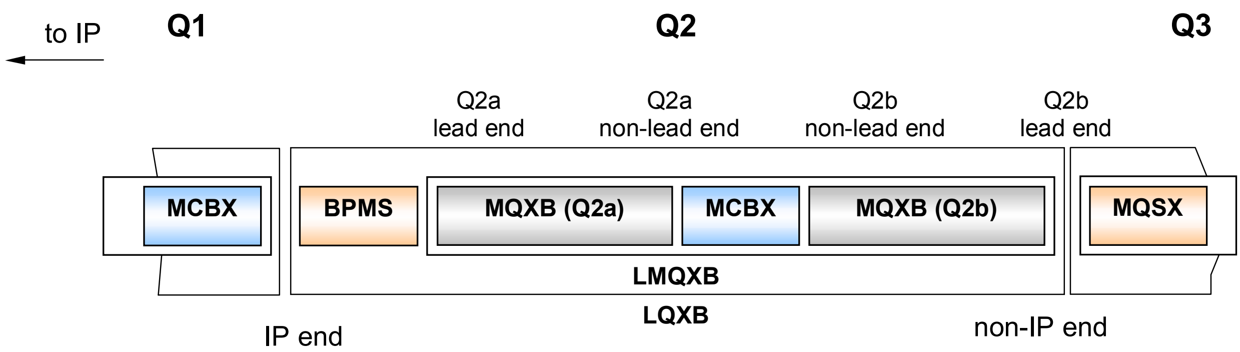

Figure 3.2. Schematic composition and location of the lead and IP ends for the LQXB cryo-assembly (Q2), from [1].

3.1 SURVEY DATA FOR Q2

The reference frame for survey data is the triad RST shown schematically in Fig. 3.3. The plane RS is the *magnetic midplane* of the magnet (defined later in this section). The S axis is the *average magnetic axis* of the magnet (defined later in this section), pointing in the direction from the lead end of Q2b to the lead end of Q2a. The R axis is oriented to the right when looking at the magnet from the lead end of Q2b. This direction corresponds to the inwards direction of the LHC (pointing towards the centre of the accelerator) for the Q2 magnets installed on the *left* of the IP in the sense of Beam 1 (turning clockwise in the LHC). The T axis is normal to the RS plane, and is oriented so that the triad RST forms a right-handed reference frame.

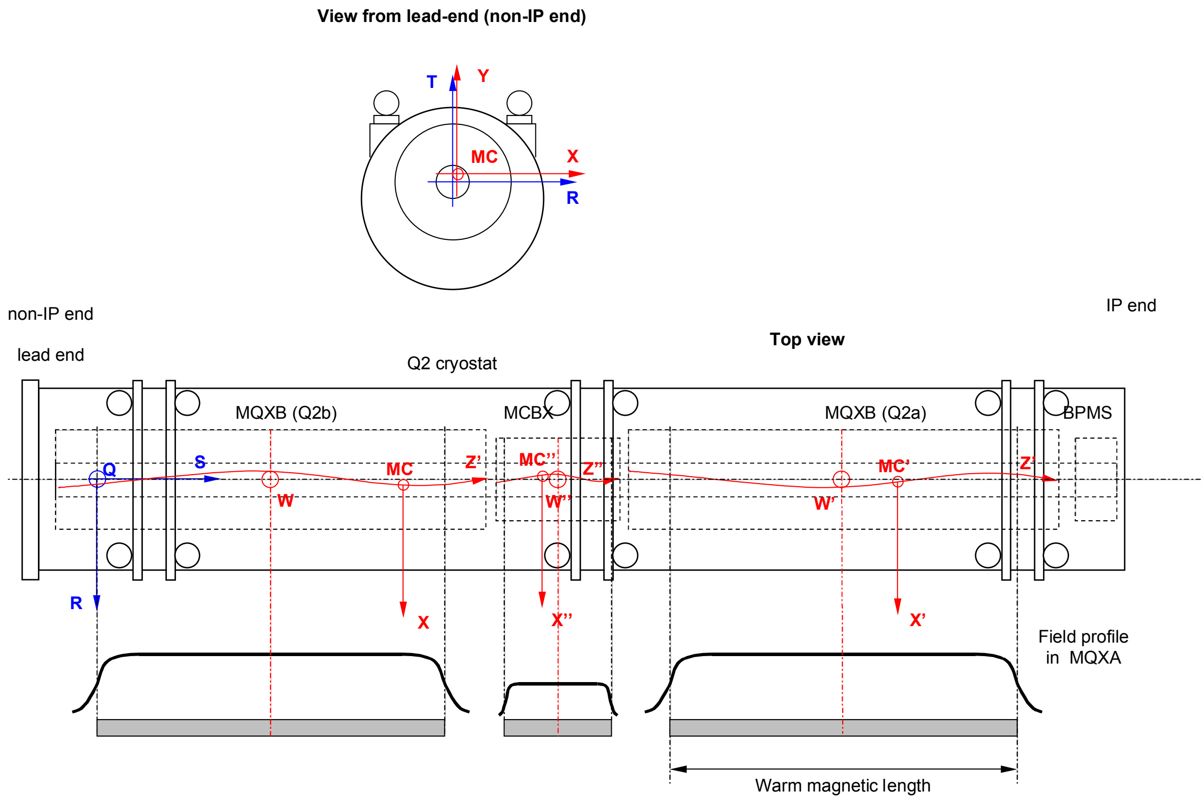

Figure 3.3. Definition of the reference frame for magnetic and alignment data of Q2. The fiducials positions on the cryostat are shown schematically. The position of points Q, the origin of the RST frame, W, W' and W'', the origins for magnetic data, is determined in warm conditions.

The *average magnetic axis* is the weighted average of the locus of the centers of the main quadrupole field (magnetic centers) of the assembly formed by the two MQXB quadrupoles. The weight for the average is the local quadrupole field strength. The average magnetic axis is hence the line along which the integrated skew and normal dipole from MQXB (Q2a) and MQXB (Q2b) is zero.

The *magnetic midplane* is defined as the plane normal to the average field direction of the assembly formed by the two MQXB quadrupoles, and containing the magnetic axis S. The average field direction is defined in Appendix II

The origin of the RST frame is located at the point Q that corresponds to the beginning of the *magnetic length* of the MQXB (Q2b) element, on the lead end, in warm conditions. The magnetic length is defined in Appendix II.

The survey data consists in a set of coordinates of the Q2 magnet fiducials (located on the cryostat) in the RST reference frame. The format of this data is defined in [9].

Warm-to-cold correlation for the survey data is defined as for Q1 through the change in the position of the fiducials in the RST frame. It is planned to measure the warm-to-cold geometry change for all the magnets.

3.2 MAGNETIC DATA FOR Q2

3.2.1 MQXB QUADRUPOLE

The reference frame for magnetic data of the two MQXB quadrupoles in Q2 is defined using the same convention as for Q1. The definition is reported schematically in Fig. 3.3. The magnetic data consists of local and integrated main quadrupole strength and transfer function, magnetic length, local and average field direction, local and average higher order harmonics, local and average magnetic center coordinates and the longitudinal distance between the quadrupoles, defined as the distance between the longitudinal magnetic centers W and W', and measured along the S axis.

3.2.2 MCBX CORRECTOR

The reference frame for magnetic data of the MCBX corrector package in Q2 is defined as for Q1, and is shown schematically in Fig. 3.3. The magnetic data for the MCBX corrector consists in the integrated field strength of the two windings B_1dl and A_1dl, the magnetic length of the two windings L_{mB1} and L_{mA1}, the average field direction of the two windings $<\alpha_{1B1}>$ and $<\alpha_{1A1}>$, and the longitudinal position of the magnetic element with respect to the main quadrupole MQXB (Q2b), i.e. the distance between the longitudinal magnetic centers W and W'', measured along the S axis. (assumed to be identical for the two corrector windings).

4. Q3

The Q3 (LQXC) optical element contains a quadrupole MQXA (the same as in Q1) whose cross section is shown in Fig. 2.1.

Q3 is equipped with a MQSX corrector package (on the lead-end side) and a MCBXA corrector package (on the non-lead end side). A combined, non-linear correction coil package MCSOX is mounted adjacent to the MCBXA. The lead end of Q3 is located on the IP side of the cryo-assembly. See Fig. 4.1 for a schematic of the Q3 configuration and the location of the lead and IP ends.

Similar to Q1, the cryostat of Q3 is equipped with eight fiducial nests for Taylor-Hobson fiducial spheres. Four of these fiducials are used for alignment, while the other four will house instrumentation for monitoring the position of the cryo-assembly after installation.

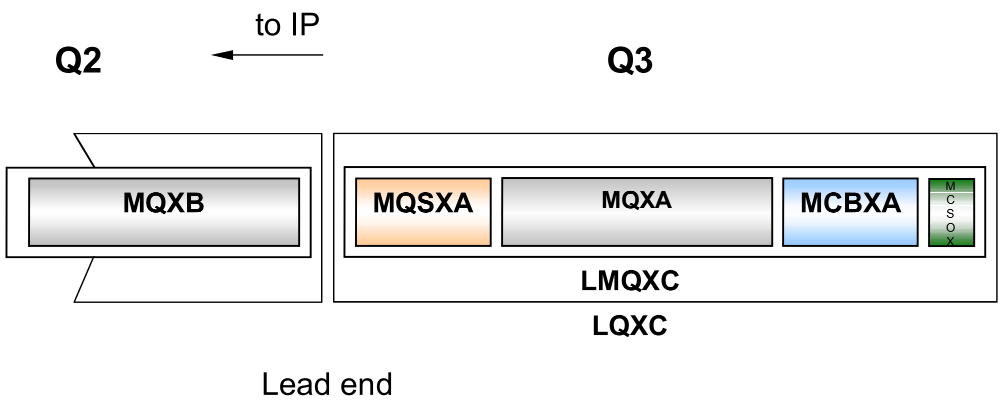

Figure 4.1. Schematic composition and location of the lead and IP ends for the LQXC cryo-assembly (Q3), from [2].

4.1 SURVEY DATA FOR Q3

The definition of the survey frame for the fiducials on Q3 is the same as for Q1. The definition is reported schematically in Fig. 4.2. As for Q1, the R axis is oriented to the right when looking at the magnet from the lead end. This direction however corresponds to the inwards direction of the LHC (pointing towards the centre of the accelerator) for the Q3 magnets installed on the *right* of the IP in the sense of Beam 1 (turning clockwise in the LHC).

The survey data consists in a set of coordinates of the Q3 magnet fiducials (located on the cryostat) in the RST reference frame. The format of this data is defined in [9].

The warm-to-cold correlation is defined as for Q1 and Q2, as the change in the position of the fiducials in the RST frame. Similar to Q1, cold tests are planned only on a sample of the Q3 production, and the correlation will hence be based on measured and computed values.

4.2 MAGNETIC DATA FOR Q3

4.2.1 MQXA QUADRUPOLE

The reference frame for magnetic data of the MQXA quadrupole in Q3 is defined using the same convention as for Q1. The definition is reported schematically in Fig. 4.2. The magnetic data consists of local and integrated main quadrupole strength and transfer function, magnetic length, local and average field direction, local and average higher order harmonics, local and average magnetic center coordinates. The same definitions as for Q1, as reported in Appendix II, apply also to Q3.

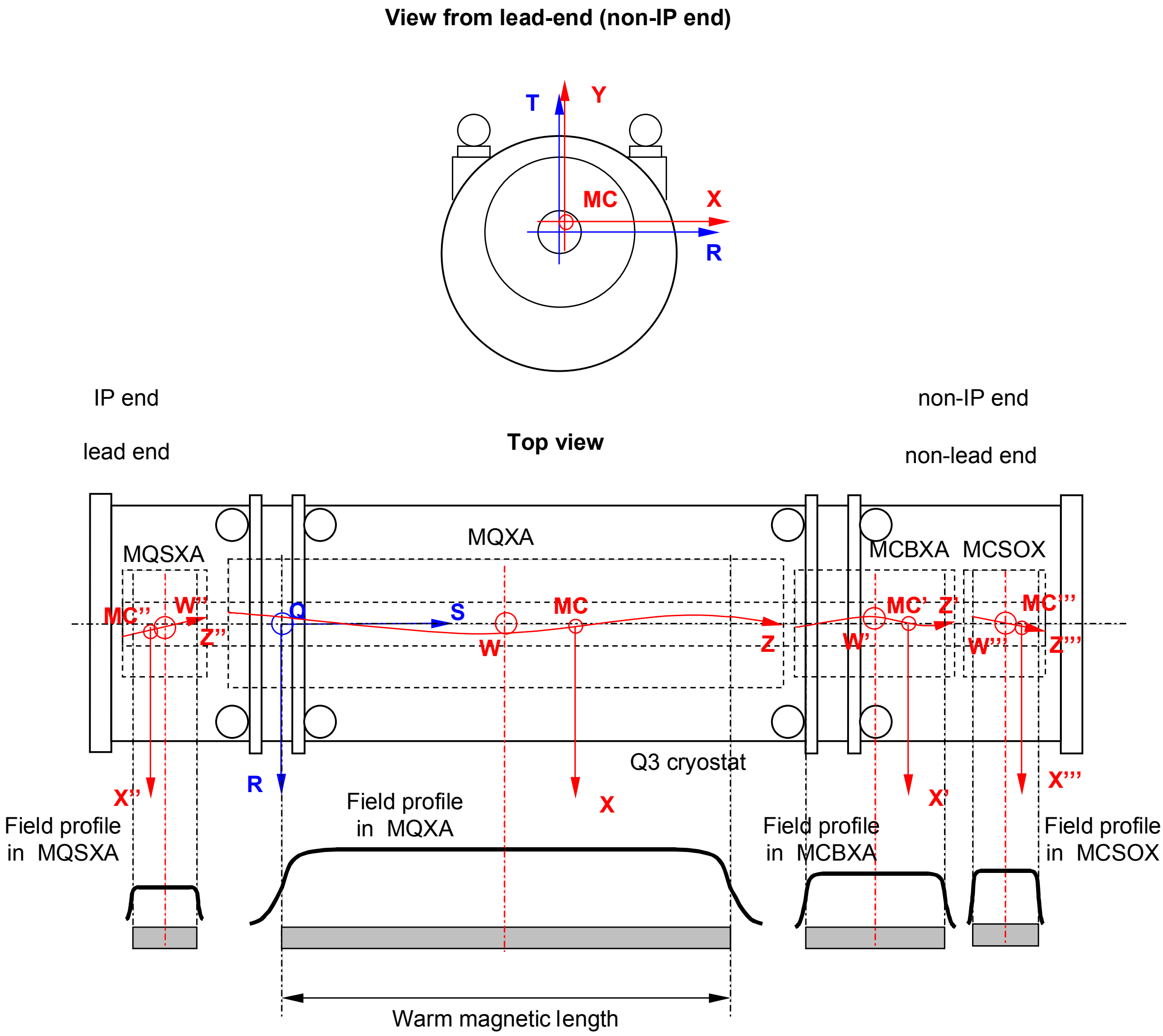

Figure 4.2. Definition of the reference frame for magnetic and alignment data of the Q3 quadrupoles. The fiducials positions on the cryostat are shown schematically. The position of point Q, the origin of the RST frame, and of the points W, W′, W′′ and W′′′, the origins of the magnetic frames, is determined in warm conditions.

4.2.2 CORRECTOR PACKAGES

The MCBXA corrector package contains two dipole windings (as for the MCBX) and additional nested corrector windings (b3 and b6). The co-axial windings are assumed to be aligned longitudinally and transversally in accordance with the manufacturing drawings, and the only error considered is a relative angle misalignment among the windings. The reference frame for magnetic data of the MCBXA corrector package is the triad X′Y′Z′ shown in Fig. 4.2. The Z′ axis is the *magnetic axis* (the locus of the magnetic centers MC′) of the lowest order corrector winding (b3) inserted in the MCBXA. The X′ and Y′ axes are respectively parallel and normal to the RS plane.

The origin of the X′Y′Z′ frame is located in the longitudinal magnetic center W′ of the dipole corrector windings (assumed to be longitudinally centered by construction).

The MQSX corrector package contains a skew quadrupole corrector. The reference frame for magnetic data of the MQSXA corrector packages is the triad X′′Y′′Z′′ defined analogously to X′Y′Z′ and also shown in Fig. 4.2. The Z′′ axis is the *magnetic axis* of the skew quadrupole corrector winding. The X′′ and Y′′ axes are respectively parallel and normal to the RS plane.

The origin of the X′′Y′′Z′′ frame is located in the longitudinal magnetic center W′′ of the skew quadrupole corrector.

The reference frame X′′′Y′′′Z′′′ of the MCSOX corrector package is defined by analogy with the above definitions, and is shown in Fig. 4.2. The Z′′′ axis is the *magnetic axis* of the skew sextupole corrector winding. The X′′′ and Y′′′ axes are respectively parallel and normal to the RS plane. The origin of the X′′′Y′′′Z′′′ frame is located in the longitudinal magnetic center W′′′ of the skew sextupole corrector.

The magnetic data for the corrector packages consist in the integrated field strength $B_n$dl and $A_n$dl of each corrector of order n, the magnetic length, the average field direction $<\alpha_{nBn}>$ and $<\alpha_{nAn}>$ of each corrector winding, the average magnetic center coordinates Δx, Δy and the longitudinal position of the magnetic element with respect to the main quadrupole MQXA.

The average field direction for each corrector winding, $<\alpha_{nBn}>$ and $<\alpha_{nAn}>$ for normal and skew correctors, is reported in the X′Y′ reference frame for the MCBXA, in the X′′Y′′ reference frame for the MQSX, and in the X′′′Y′′′ reference frame for the MCSOX, i.e. with respect to the normal T to the magnetic midplane of the MQXA quadrupole.

The average magnetic center coordinates are given in the RST frame, i.e. with respect to the axis of the MQXA quadrupole.

The longitudinal position of the MCBXA, MQSX and MCSOX corrector packages is defined as the distance between the longitudinal magnetic centers W′, W′′ and W′′′, and the magnetic center of the main quadrupole MQXA measured along the S axis.

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[16] L. Deniau, L. Bottura, Table F Int Harm, available at the URL: http://www.cern.ch/m/mtauser/www/html/DAP/f_int_harm.html.

6. APPENDIX I – EXPANSION OF THE MAGNETIC FIELD IN THE COMPLEX PLANE

We recall the definition [10], [11] of the harmonic expansion of the magnetic field $\boldsymbol{B}$ in the 2-D imaginary plane (x,y) in terms of the complex variable $\mathbf{z}=x+i\,y$:

$$\boldsymbol{B}(\mathbf{z}) = \mathbf{B}_y + i\mathbf{B}_x = \sum_{n=1}^{\infty} \mathbf{C}_n \left(\frac{\mathbf{z}}{R_{ref}} \right)^{n-1}$$

where the coefficients $\mathbf{C}_n$ appearing above are the complex harmonic coefficients, and R_{ref} is the reference radius (R_{ref} is 17 mm for LHC).

7. APPENDIX II – DEFINITIONS FOR INTEGRAL AND AVERAGE FIELD VALUES

7.1 INTEGRATED QUADRUPOLE STRENGTH

The integrated quadrupole strength B_2dl is defined as the integral of the local quadrupole strength along the length of the magnet, including the full contribution of the magnet ends:

$$B_2 dl = \int_{L_{magnet}} B_2(Z)dZ$$

This integral generally translates in a sum of measured values over adjacent lengths, e.g. obtained with short rotating coils, but can also be obtained directly from an integral measurement, e.g. with a stretched wire or a long rotating coil. The integration formula does not take into account the possible variations in the quadrupole field direction along the magnet length. This effect is negligible for variations of the local field direction below 5 mrad and was not taken into account for simplicity.

7.2 QUADRUPOLE MAGNETIC LENGTH

The magnetic length is defined as the ratio of the integrated quadrupole strength B_2dl to the average quadrupole strength in the body of the magnet (i.e. excluding end effects):

$$L_m = \frac{B_2 dl}{\dfrac{\int_{L_{body}} B_2(Z)dZ}{L_{body}}}$$

The magnetic length corresponds to the length of an ideal magnet with the same integrated strength as the real magnet considered and a *window-frame* field profile, with field strength identical to the body average strength.

7.3 INTEGRATED QUADRUPOLE TRANSFER FUNCTION

The integrated transfer function T_2dl is defined as the ratio of the the integrated quadrupole strength to the operating current:

$$T_2 dl = \frac{B_2 dl}{I}$$

The integrated transfer function is given in units of [T m @ R_{ref}/A].

7.4 AVERAGE QUADRUPOLE FIELD DIRECTION

The average quadrupole field direction $<\alpha_2>$ is given by the weighted integral of the local field direction along the magnet length. The weight is the quadrupole field strength:

$$<\alpha_2>=\frac{\int_{L_{magnet}} \alpha_2(Z)B_2(Z)dZ}{\int_{L_{magnet}} B_2(Z)dZ}$$

The average quadrupole field direction $<\alpha_2>$ is given in units of [radians].

7.5 AVERAGE QUADRUPOLE HARMONICS

The average higher order harmonics of order n, normal $<b_n>$ and skew $<a_n>$ are computed similarly to the average quadrupole field direction:

$$<b_n>=\frac{\int_{L_{magnet}} b_n(Z)B_2(Z)dZ}{\int_{L_{magnet}} B_2(Z)dZ}$$

$$<a_n>=\frac{\int_{L_{magnet}} a_n(Z)B_2(Z)dZ}{\int_{L_{magnet}} B_2(Z)dZ}$$

The average harmonics b_n and skew a_n are given in relative units of 10^{-4} of the main field strength at the reference radius.

7.6 INTEGRATED AND AVERAGE FIELD FOR CORRECTOR MAGNETS

The definition of the integrated and average field quantities for the corrector magnets follows by analogy the definitions above, substituting the main field of the corrector B_n or A_n and the local field direction of the corrector α_{nBn} or α_{nAn} for the main quadrupole field B_2 and the main quadrupole field direction α_2 respectively.